Math 2A Quiz 1 Version 2 **NAME YOU ASKED T**

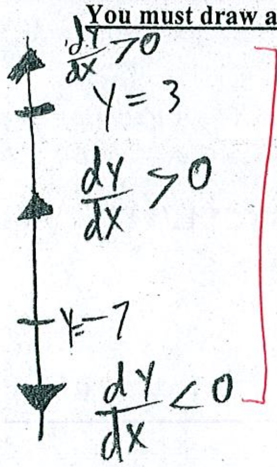
Fri Oct 6, 2017

SCORE: 20 **/ 30 POINTS** 17+3

1. **NO CALCULATORS OR NOTES ALLOWED**
2. **UNLESS STATED OTHERWISE, YOU MUST SIMPLIFY A**
3. **SHOW PROPER CALCULUS LEVEL WORK TO JUSTIFY YOUR ANSWERS**

Consider the DE $\frac{dy}{dx} = (7+y)^3(3-y)^2$. $Y=-7$ $Y=3$ **SCORE:** 4 **/ 6 PTS**

[a] Find all equilibrium solutions of the DE and classify each as stable, unstable or semi-stable.
You must draw a phase portrait to get full credit.

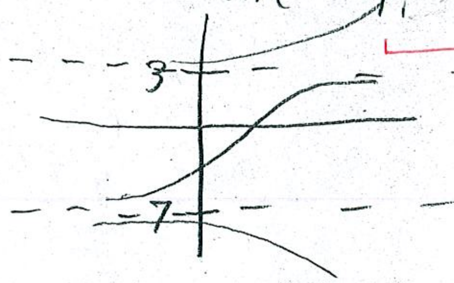


(2)

$\frac{dY}{dX} = 0$ when $Y = -7, 3$ of the DE

equilibrium solution are $Y = -7$, $Y = 3$

(1/2) (1/2)



[b] If $y = m(x)$ is a solution of the DE such that $m(-8) = -5$, what is $\lim_{x\to\infty} m(x)$?

$$\boxed{\lim_{X\to\infty} m(X) = 3}$$

(1)

An object of mass m kilograms is shot upward at high speed. Its velocity is affected by the forces of gravity and air resistance. Because of the high speeds involved, the force of air resistance is proportional to the square of the velocity. (Assume $v > 0$ corresponds to upward motion.) **SCORE:** 3 **/ 4 PTS**

Write a differential equation for the velocity of the object as it rises. All symbolic constants in your differential equation must represent positive numbers. You may use the symbolic constant g to represent $9.8\,\frac{m}{s^2}$. **Do NOT use the absolute value function in your answer.**

(1) $m\left(\frac{dV}{dt}\right) = mg - kV^2$ (2)

if downwards is positive

$$\boxed{\frac{dv}{dt} = g - \frac{kV^2}{m}}$$

where k is some constant

What does the Existence and Uniqueness Theorem tell you about possible solutions to the IVP $x+(y')^3 = 1+\sin y, \quad y(1)=\pi$? **Justify your answer properly, but briefly.**

SCORE: 0 / 4 PTS

$$(Y'(x))^3 = \frac{1+\sin Y}{X} \rightarrow Y' = \sqrt[3]{\frac{1+\sin Y}{X}}$$

$$f(X,Y) = Y' = \frac{(1+\sin Y)^{1/3}}{X^{1/3}}$$

$$\frac{\delta f}{\delta Y} = X^{-1/3}\left(\frac{1}{3}(1+\sin Y)^{-2/3} \cos Y\right)$$

$\frac{\delta f}{\delta Y}$ is continuous everywhere except at $Y = \frac{3\pi}{2}$

$\lim\limits_{X \to \infty} f(X,Y)$ is continuous everywhere except at $X=0$

Consider the IVP $y' = 10xy + 25x, \quad y(1) = -2$. Use Euler's method with $h = 0.2$ to estimate $y(1.4)$.

SCORE: 4 / 4 PTS

$$Y(1.2) = Y(1) + Y'(1)(0.2) \qquad Y'(1) = 10(1)(-2)+25 = 5$$

$$Y(1.2) = -2 + 5\left(\frac{1}{5}\right) = -1 \quad (1) \quad (1)$$

$$Y(1.4) = Y(1.2) + Y'(1.2)(0.2) \qquad Y'(1.2) = 10(1.2)(-1) + 25\left(\frac{6}{5}\right) = -12+30 = 18$$

$$Y(1.4) = -1 + 18\left(\frac{1}{5}\right) \quad (1)$$

$$= -1 + \frac{18}{5} = -\frac{5}{5} + \frac{18}{5} = \frac{13}{5} = \boxed{2.6} \quad (1)$$

Determine if $y = Ax + B\ln x + \frac{1}{x}$ is a family of solutions of the DE $(x^2 - x^2\ln x)y'' + xy' - y = \frac{2\ln x}{x}$.
State your conclusion clearly.

SCORE: 6 / 6 PTS

$$Y' = A + \frac{B}{X} - \frac{1}{X^2} \qquad Y'' = -\frac{B}{X^2} + \frac{2}{X^3} \quad (1)$$

$$(1) \; X^2(1-\ln X)\left(-\frac{Bx+2}{X^3}\right) (1) + X\left(A + \frac{B}{X} - \frac{1}{X^2}\right) - AX - B\ln X - \frac{1}{X}$$

$$= (1-\ln X)\left(-B + \frac{2}{X}\right) + AX + B - \frac{1}{X} - AX - B\ln X - \frac{1}{X}$$

$$= \cancel{-B} + \frac{2}{X} + \cancel{B\ln X} - \frac{2}{X}\ln X + \cancel{B} - \frac{1}{X} - \cancel{B\ln X} - \frac{1}{X} = -\frac{2}{X}\ln X + \cancel{\frac{2}{X}} - \cancel{\frac{2}{X}}$$

$$= -\frac{2}{X}\ln X \quad (2)$$

No $Y = Ax + B\ln X + \frac{1}{X}$ is not a family of solutions of the DE (1)